Physics EE on Aerodynamics

What is the relationship between Frequency and surface area against lift of the propeller of a toy helicopter?

Word count: 2777

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Introduction

When I was young, my father bought me a toy helicopter from his business trip to London. He taught me how to play with the helicopter and we spent a lot of time playing with it and I truly enjoyed that time. Recently, I came across a father and a boy playing with a toy helicopter like I had done when I was young. However, I found out their helicopter was much faster and flying higher than my memory. This made me to think "what is the relationship between lift and frequency as well as lift and surface area of the propeller of a toy helicopter?" As a result, I decided to make this my EE topic and find out what made difference. Using a toy helicopter and altering the rotational speed of blades and lengths of blade, I will find out the relationship between length/rotational speed of blades and the lift it provides to the helicopter.

Theory

When a helicopter leaves the ground, there are four forces acting on the helicopter: thrust, drag, weight and lift. Thrust is the force produced by the propeller and this is the force acts parallel to the longitudinal axis resulting in pushing the helicopter to the front. Drag is the force opposing thrust. This force is caused by air resistance caused by airflow by the body of the helicopter. Lift, the force I am focusing on in this EE, is produced by the propeller and acts perpendicularly up to the moving direction. Weight, which is the sum of every load in the helicopter, pulls the helicopter down. Obviously, only when lift is bigger than the weight, helicopter will fly. To achieve this, the blade of helicopter has special shape. When it rotates, the upper part of the blade receives less pressure compared to the bottom part of the blade resulting in as lift.

This is the equation regarding rotational frequency and the radius of the blades.

$f^2 = \frac{F}{8\pi^2 \rho \lambda^2 R^2}$ (LieblMichael) *f* Stands for the rotational frequency of the propeller. Frequency is measured in hertz (Hz). The lift (F) is the force required to lift a helicopter and measured in (N) the density of air is written as ρ and unit is (kgm^3). The $8\pi^2$ is the constant. The Lambda, λ, is a parameter called the rotor inflow ratio. Finally R stands for radius of the blade. To prove that equation, I did two experiments. In one experiment, I changed the frequency and in another one, I changed the radius of the blade. Also to make my calculation easier, I changed F into mg using Newton's second law. As a result, this equation shows frequency squared is inversely proportional to the radius squared.

I considered gravity as 9.8 kg^{-1} and air density as 1.2kg^{-1} (ShelquistRichard). The helicopter alone was 66g and the mass I used was 200g.

Hypothesis

As the equation above shows lift force will increase if either radius of the blade or rotating frequency increases. In my experience, I would expect when frequency or radius increases the lift force also increases. According to the equation frequency squared and lift mass will have linear relationship and radius quadrupled with lift will have linear relationship.

Background information

The first helicopter was designed by Leonardo da Vinci in 1486. However as the most inventions he thought about, he could not actually made it because of power issue. Many people tried to build a working helicopter for long time but the first working helicopter was built in 1907 by Paul Cornu. However to build the one could be stably controlled took many years. When it was fully developed, it was used as a military purpose from World War 2. The way helicopter gain lift is different from airplane. When the blades of the propeller rotate, it pushes the air downwards resulting as a lift and it is obvious that when the frequency rises, the force it experiences also gets higher. When the lift and the gravity are balanced, helicopter can hover in a fixed spot. I also found out that when helicopter is on the ground, it will experience more lift because the air pushed down by the blade will bounce against ground. However, if there is only one set of blade spinning, the helicopter will also spin in the opposite direction to make the total angular momentum zero. To solve this problem, helicopters have stabilizer rotors on the tail to provide opposite torque to prevent body of the helicopter to rotate. Some big helicopters have two sets of main propeller spinning in different direction for stabilization purpose. The helicopter I used in this experiment has two main sets of blades. During the experiment I ignored the stabilizer wing.

Variables

Variable name	Variable Type	How is it controlled/measured
Rotational frequency of helicopter blade	Independent	By using a helicopter controller and stroboscope
Length of the blade of the propeller	independent	By cutting in same interval. Using a ruler.
Lift force	dependent	By fixing helicopter to a scale and reading the difference
Mass of helicopter	Control	Using a scale and making sure the first point is same.

Equipment

1. Toy helicopter
2. Electric balance (up to 1kg)
3. Stroboscope (up to 2000 rpm)
4. 200g mass

When I used the balance during the experiment, I found out that the mass was constantly changing. In my opinion, this was because of the vibration of the motor it had and because of the airflow it had. As a result, I only used the constant reading which was up to 0 decimal places. Hence my lift mass uncertainty is ±0.5g. Also, the stroboscope I used to measure frequency was not digital. As a result, I hand to manually find out the frequency by increasing frequency by 1 rpm and see rather the blade seems not moving. Also, since the equation was using Hz as a unit, I had to convert RPM into Hz by dividing RPM by 60. Hence my frequency uncertainty is ±0.017 Hz.

Here's a picture of my equipment

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First experiment: to find out the relationship between rotational frequency and lift.

Procedure

1. Glue the helicopter to the balance and make sure that the helicopter does not move or vibrate even when the motor is spinning in full speed. This will make the experiment more accurate by excluding other factors to affect the reading of the balance.
2. Set the scale to show zero when there is helicopter on, and add a 200g mass to the scale and make sure the scale is showing 200.
3. At this point, it should look like this

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4. Using a remote controller, start the motor.
5. The radius of the propeller should be 105mm.
6. Start from the lowest frequency possible and slowly increase the frequency.
7. Using a stroboscope, measure the frequency and record current mass

displayed on the scale.

8. Current frequency can be measured by using a stroboscope. When the propeller looks like it is not moving under stroboscope's light, it is that frequency.
9. Slowly increase the frequency of motor till it reaches the maximum frequency.
10. Record the frequency and mass shown about each frequency on the scale at that point.
11. Repeat from the lowest frequency.

Safety

While using a glue to stick the helicopter to the scale, make sure you wear gloves so you do not have physical contact with glue.

When the blade of the propeller is spinning, do not touch or get near to the helicopter. There is a risk of getting hurt because of spinning blades

Data of first experiment

Orginal mass(g) (±1)	Scale mass(g)(±1)	Change in mass(g)(±1)	Lift(N)	Frequency(Hz)(±0.01)	$f^2(Hz^2)$(±0.01)
200	160	40	0.392	21.58	465.84
200	158	42	0.412	21.83	476.69
200	158	42	0.412	22.08	487.67
200	157	43	0.421	22.35	499.52
200	156	44	0.431	22.18	492.10
200	155	45	0.441	22.67	513.78
200	154	46	0.451	22.68	514.53
200	154	46	0.451	23.23	539.79
200	153	47	0.461	23.12	534.38
200	152	48	0.470	22.95	526.70
200	151	49	0.480	23.78	565.65
200	149	51	0.500	23.78	565.65
200	146	54	0.529	24.15	583.22
200	145	55	0.539	24.25	588.06
200	144	56	0.549	23.95	573.60
200	143	57	0.559	24.58	604.34
200	142	58	0.568	24.77	613.39
200	138	62	0.608	25.20	635.04
200	137	63	0.617	24.50	600.25
200	135	65	0.637	25.20	635.04
200	134	66	0.647	25.58	654.51
200	130	70	0.686	24.78	614.21
200	130	70	0.686	24.85	617.52
200	129	71	0.696	26.35	694.32
200	126	74	0.725	26.67	711.11

Sample Results

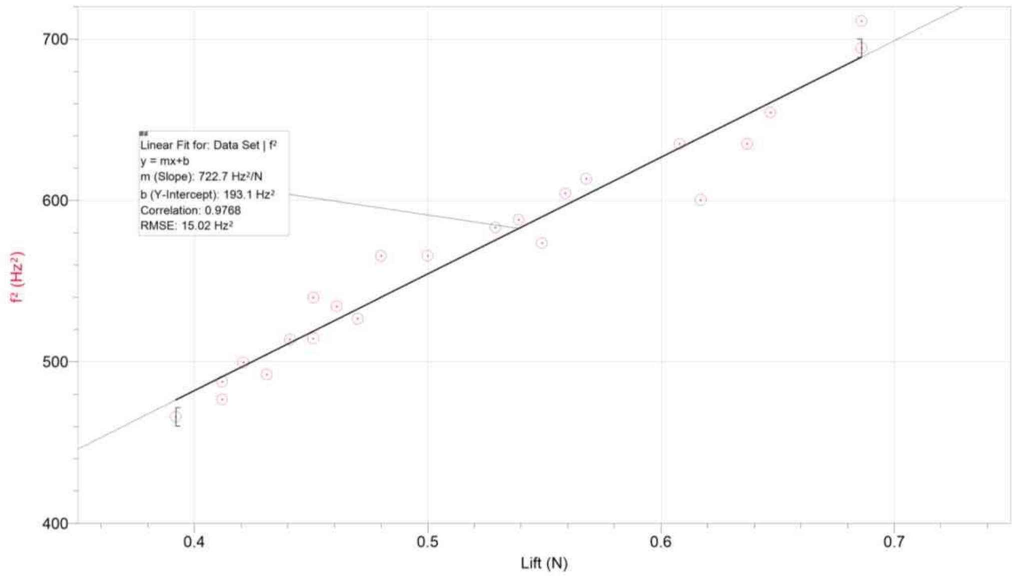
Change in mass: (Original mass – Scale mass) e.g.: 200-160=40

Lift: (Change in mass/1000*9.8) e.g.: 40/1000*98=0.392

F^2: Frequency^2 e.g.: 21.58^2=456.84

The original reading was 200g and it was the constant value. The propeller was started at the lowest possible frequency 21.58Hz. Corresponding value was calculated and lift was calculated by subtracting the scale mass from the original mass. For lower frequency the less lift the helicopter experienced. Since there is only two forces acting on the helicopter, mass and the thrust the propeller provides, and mass of the helicopter did not change, all of this change in lift is due to the rotation of the propeller. When blades spin faster, it will give more push downward resulting in more force upwards.

To draw the graph, I had to square the frequency. Also I only need lift mass so I did a calculation and added two parts to my results table.



Using the data above, this is the graph I got. The graph clearly shows that the square of frequency and lift mass have linear relationship. The line of best fit has correlation of 0.98 which is quite similar to 1. Also, the Y intersection is 193.1 since this value is quite similar to the original value 200, I can say that this experiment has minor uncertainties but no huge errors. Considering these two, I can prove that the relationship of f^2 and lift mass is proportional.

During this experiment there are some possible errors. Random errors can occur when remote control does not send signal correctly or when the scale itself does not perform correctly resulting in showing wrong mass and the stroboscope I used does not automatically show the frequency which may resulting in different frequency from what it is. Systematic error can occur because of the scale I used may mall function.

Evaluation of Experiment 1

According to the equation, $f^2 = \frac{F}{8\pi^2 \rho \lambda^2 R^2}$, the line of the best fit should cross 0,0. However, my line of the best fit crosses 0,193, which is a systemic error. The gradient I found, 722.7, is a value of $\frac{1}{8\pi^2 \rho \lambda^2 R^2}$. Since I know all the values except λ which is rotor inflow ration, I can find out what is the rotor inflow ratio. The equation is $\frac{1}{8\pi^2 \rho R^2 722.7} = \lambda^2$. ρ equals 1.2 and R equals 105 which makes the equation $\sqrt{\frac{1}{8\pi^2 1.2 * 105^2 722.7}} = 3.64 \times 10^{-5}$. The real rotor inflow ratio cannot be found on the internet but in the second experiment I will find out rotor inflow ratio and compare it to verify my experiment.

Second experiment: to find relationship between radius of the blade and lift.

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Procedure

1. Equipment and the starting conditions of the experiment should not change from the first experiment
2. Set the stroboscope to 1400 RPM and flash it to the propeller. When the propeller looks like not moving, keep it.
3. Mount the blades to the helicopter and make it spin to the decided frequency and record the mass shown on the scale.
4. Completely stop the motor and repeat for three times

5. Change the length of the blade by cutting it 1 cm at each time and measure how much lift does it provides.
6. When the length of the blade gets to 55mm, stop. This is because when the wing is shorter than 55mm, it does not make any difference.

Safety

When you are using scissor to cut the blades, be careful that you do not cut yourself.

When the blade of the propeller is spinning, do not touch or get near to the helicopter. There is a risk of getting hurt because of spinning blades

Data of my second experiment

Radious(mm) (±1)	mass(g)(±1)	change in mass(g)(±1)	average change in mass(g)(±1)	Average lift (N)	absolute error(g)(±0.1)	percentage error (%)(±0.01)	R^2(mm)(±1)
105	158	42	41.67	0.408	0.5	1.20	11025
	158	42					
	158	41					
95	170	30	30.33	0.297	0.5	1.65	9025
	169	31					
	170	30					
85	180	20	20.33	0.199	0.5	2.46	7225
	180	20					
	180	21					
75	183	17	16.67	0.163	0.5	3.00	5625
	183	17					
	183	16					
65	188	12	12.33	0.121	0.5	4.05	4225
	188	13					
	188	12					
55	192	8	7.67	0.075	0.5	6.52	3025
	193	7					
	192	8					

Sample Results

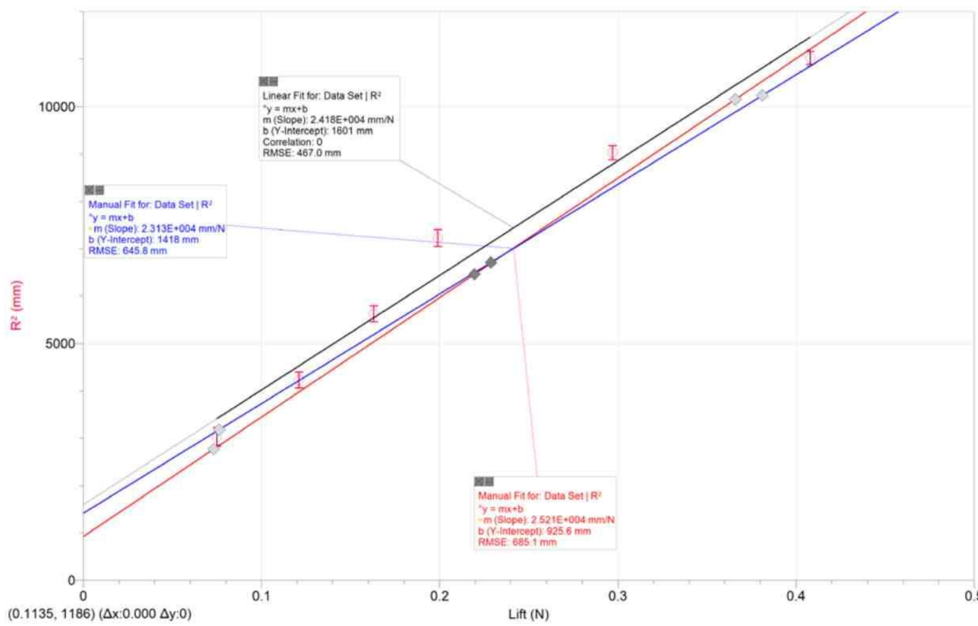
Average change in mass: (sum of masses/number of masses) e.g.: (42+42+41)/3=41.67

Average Lift: (Average change in mass/1000*9.8) e.g.:41.67/1000*9.8=0.408

Absolute error: ((max-min)/2) e.g.: (42-41)/2

Percentage error: (absolute error/average change in mass*100) e.g.: 0.5/41.67*100

I measured the lift for each radius three times. Based on the data, I found absolute error and percentage error. Absolute error of every data was 0.5 gram. Unlike the first experiment, frequency of the rotation stayed constant as 23.3 Hz. According to the theory, there is a liner relationship between radius squared and lift. As a result, I modified the data and added this part to my result table.



This is the graph I got when I plot a graph of radius of blades and average lift it provided to the helicopter. The graph shows correlation of 0.99 which proves that the radiuses quadrupled and lift it provides to the helicopter has 1 to 1 relationship thus proves the equation I based my experiment is valid. In this experiment, error ranged from 1.2% to 6.5%. This shows that this experiment is valid to the certain point but has some possible errors.

This experiment might have sources of random or systemic errors. Random error might occur because of malfunction of helicopter motor resulting in less power it should provide or more power. Also, there is a possibility that the frequency of the propeller spinning is not consistent for the whole time. Systematic error can occur because when I was using a stroboscope, I might have miss measured the frequency resulting in more or less frequency than it should have been.

Evaluation of Experiment 2

According to the equation I used, this graph should have a linear relationship with line of best fit crossing 0, 0. However, this graph does have a line of best fit with straight line but does not cross 0, 0 but cross 0, 1601 which is a systemic error. This graph has gradient of 2.418*10^4 and according to the original equation, this gradient is$8\pi^2\rho\lambda^2 f^2$. I already know ρ (1.2) and f (23.3). λ equals $\sqrt{\frac{8\pi^2 1.2*23.3^2}{2.418*10^4}}$ which is 1.458.

Conclusion

In theory, both experiments should show equal rotor inflow ratio because it is a constant value. However, in my experiment, it did not show the same value. This might be because since I had a quite a big systemic error.

Evaluation

The results obtained from the experiment are quite accurate. This is because the lift it provided in the similar frequency is all in the similar range. Also in the surface area experiment the lift it provided had ±1 gram of difference. Since the uncertainty of the electric scale was ±1 gram this is acceptable value. Finally in the whole experiment uncertainty percentage did not exceed 10%. However I found out that the experiment could be more accurate in some way.

- Despite my attempt to make the frequency of the blade stable as possible, I found out that the frequency decreases while time goes by. I think this is primarily because the helicopter was running from battery, as battery loses its power, the blade also slowed down. This is systematic error that must be taken into account but also impossible to improve because systematically I cannot make battery infinite.
- There could be some improvements to the equipment I used. First, the scale I used did not show any decimal points. This is systematic error which can be solved by using better equipment. Even though this might seems minimal, if I could use the scale which could show 3 decimal points, my data would me more accurate and allow me to have better comparison. Also, the stroboscope I used was not automatic. As a result, I had to manually increase RPM by 1 and see the blades seem to be not moving and record the data at that time. Since my eyesight is not always

perfect, there is a possibility of human error. Even though the results matches well with the stated results if I was able to use automatic stroboscope, my experiment would be easier and my results will be also more accurate.

- During this experiment, even though there were two sets of blades, I ignored the effect of one because I knew it was for stabilizing the body during the flight, not for the flight itself. However even though it was for stabilizing I cannot say that there is no effect on flight because it also creates downward thrust when it rotates. I think this is why there was a change in gradient in the second experiment. To fix this I could disassembled the blade from the helicopter and conduct the experiment without the stabilizer propeller.

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